Mid-Range Approximations in Sub-spaces for MDO problems including disparate attribute models

Jonathan Ollar[1], Vassili Toropov[2] and Royston Jones[1]

[1]Altair Engineering, UK
[2]University of Leeds, UK

The current work is an attempt to create an efficient MDO framework for solving optimisation problems including a series of high speed, non-linear explicit models (i.e. crashworthiness assessments) that depend only on restricted subsets of the total design variable set as well as much less complicated (i.e. linear static) models that depend on (almost) all design variables.

Responses from crashworthiness analysis can be highly non-linear and even discontinuous as some of the contacts may be active in only certain regions of the design space. A simple test case is shown in Figure 1. The left hand side load case is a cylinder impacting a metal sheet beam. The response is the resulting maximum effective plastic strain. On the right hand side is a torsion load case where the beam itself is twisted with a certain moment and the twist is measured. The design variables are the two middle panels on the top side of the beam.

The response surfaces in Figure 2 highlight the difference in complexity for the two load cases. The left hand side plot shows a highly non-linear equivalent plastic strain response resulting from the explicit non-linear impact simulation while the right hand side plot shows a smooth response surface resulting from the linear static torsion load case.

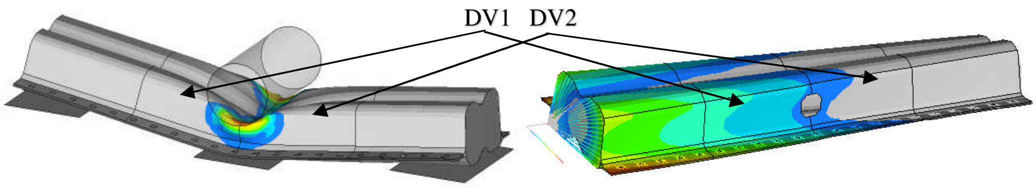

Figure 1. Left: Impacting cylinder (explicit) load case. Right: Static torsion (implicit) load case.

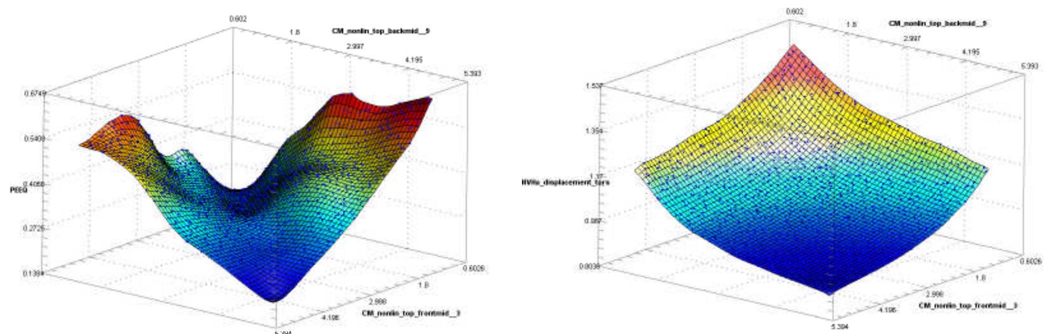

Figure 2. Left: Equivalent plastic strain (impact load case). Right: Twist (static torsion load case).

Because of the considerable disparity in complexity of the models and resulting response surfaces, one can draw the conclusion that there should be different procedures for handling the different types of models within the MDO framework. The more complex impact load case presented would need more DOE points to approximate the response behaviour than the more simple static torsion load case.

While the crash response is highly non-linear and requires a higher density test plan, in many cases they might be affected only by a subset of the total set of design variables. Consider the simple beam model in Figure 1. The static torsion subcase will be affected by (almost) the entire structure but the crash response is mostly affected by the two design variables that were chosen. By using engineering knowledge and variable screening methods, as described by Tu and Jones (2003) the dimensionality of each test plan can be brought to its minimum.

The multipoint approximation method, as reported by Toropov (1989), Toropov (1992) and Toropov et al. (1993) is an iterative optimisation technique based on mid-range approximations built in trust regions. A trust region is a sub domain of the design space in which a set of design points, treated as a small-scale design of experiments (DOE), are evaluated. These and a subset of previously evaluated design points are used to build approximations of the objective and constraint functions that are considered to be valid in the current trust region. The trust region will then translate and change size as the optimisation progress. The trust region strategy has gone through several developments to account for the presence of numerical noise in the response function values, see van Keulen et al. (1996), Toropov et al. (1996) and occasional simulation failures (also termed domain-dependent calculability of the response functions), Toropov et al. (1999). The mid-range approximations used in the trust regions, as originally suggested by Toropov (1989), are intrinsically linear functions (i.e. nonlinear functions that can be led to a linear form by a simple transformation) for individual sub-structures, and assembly of them for the whole structure. This was enhanced by the use of gradient-assisted approximations (Toropov et al., 1993), use of simplified numerical models that is also termed a multi-fidelity approach (Toropov and Markine, 1996) and the use of analytical models derived by Genetic Programming (Toropov and Alvarez, 1998). The most recent development (Polynkin and Toropov, 2012) involved the use of approximation assemblies, i.e. a two stage approximation building process that is conceptually similar to the original one used by Toropov (1989) but free from the limitation that lower level approximations are linked to individual substructures.

The moving least squares method was proposed by Lancaster *et al.* (1981) for smoothing and interpolation of scattered data and later used in the mesh-free form of the FEM (Liszka, 1984). As described by Choi et al. (2001) it can be used as a technique for surrogate modelling and used in MDO frameworks. The moving least squares method is a weighted least squares method where the weights depend on the Euclidian distance from a sample point to where the surrogate model is to be evaluated. The weight value for a certain sample point decays as the distance increases. Describing the weight decay with a Gaussian function tends to be the most useful option even though many others have been evaluated by Toropov *et al.* (2005). As demonstrated by Polynkin *et al.* (2010) the cross validated moving least squares method can be used both for design variable screening and for surrogate modelling.

In the presented research the multipoint approximation method is extended to use local test plans and moving least squares approximations built in different subspaces of the total design variable space in an attempt to create an efficient MDO framework for incorporating crashworthiness assessment in MDO.

The presentation will describe the optimization process in which to deal with the curse of dimensionality different parameterization approaches are used for the computationally heavy responses (e.g. crashworthiness-related) and the lighter responses (e.g. buckling, global stiffness, etc.). To keep such an MDO problem solvable the heavy responses are represented by a smaller number of design variables, and the lighter responses may require many more design variables. This way the approximation building DOEs are separated, i.e. have a small number of DOE points for the crashworthiness responses and many more for the lighter responses. Finally, all the sub-spaces corresponding to all the disciplines are combined into the total design variable space of the whole problem and an optimization problem is iteratively solved in each of the trust regions of the combined space.

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